

Trigonometry

The word **Trigonometry** derives from ancient Greek to describe the branch of maths that involves looking at **lengths and angles of triangles**.

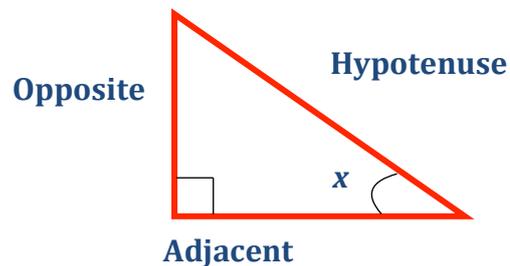
Trigonometry at KS3 and GCSE mostly looks at **right-angled triangles** - usually, you will be asked to solve one of the angles or one of the lengths of the triangle.

Labeling a Triangle

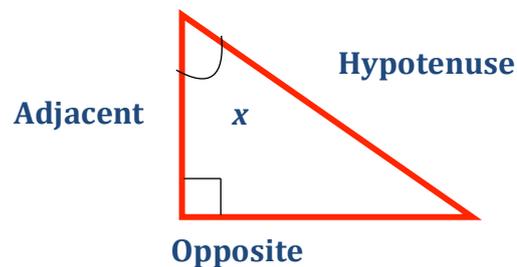
First things first, let's label a right-angled triangle. The three lengths of the triangle are called:

1. The **Hypotenuse** - the **longest side** & the side opposite the right angle.
2. The **Adjacent** - the side **next** to the angle you are trying to find.
3. The **Opposite** - the side **opposite** the angle you are trying to find.

An example of a labeled triangle is below, where x is the missing angle.



If the missing angle was the other one then the labels would look like this.



The Opposite and Adjacent have **swapped** but the Hypotenuse is the **same**.

Labeling the triangle is **very important**, we will use these labels in the next step.

Sine, Cosine & Tangent

To help us work out the missing angle or missing length of a right-angled triangle, we use some special ratios. These are:

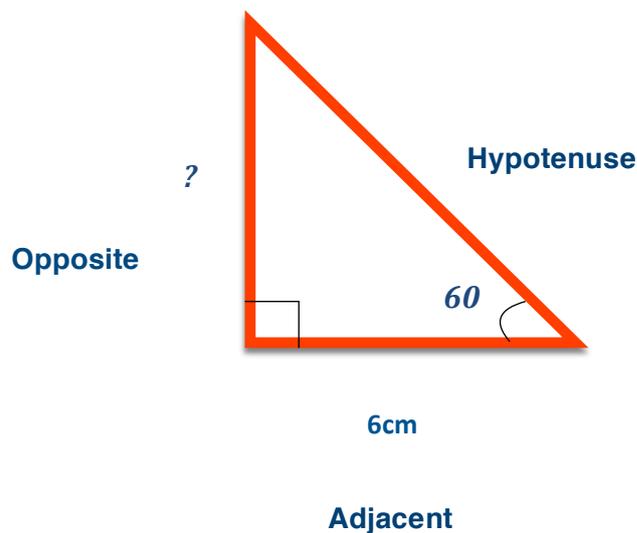
$$\tan x = \frac{\text{Opposite}}{\text{Adjacent}}$$

$$\cos x = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\sin x = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

A good way to remember these is “**SOH CAH TOA**” - Sine is **O**pposite over **H**ypotenuse, Cosine is **A**djacent over **H**ypotenuse & Tangent is **O**pposite over **A**djacent

Worked example - finding a missing side:



Step 1: Make sure you label the sides first (as above) and write out SOH CAH TOA to help you.

Step 2: Select which ratio to use (Sine, Cosine, or Tangent) based on the sides you are working with. In this case we have been given the adjacent and have been asked to find the opposite side - so we will use the Tangent ratio.

Step 3: Set up the tangent formula and then insert the numbers for this question:

$$\tan 60 = \frac{\text{Opposite}}{6}$$

Step 4: Rearrange the formula so the side you are trying to find is the subject. In this case it will be:

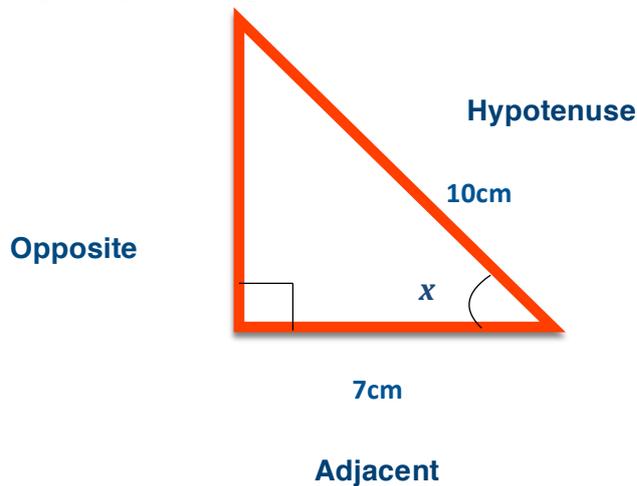
$$\text{Opposite} = 6 \times (\tan 60)$$

Then simply punch this into your calculator and we get the answer:

$$\text{Opposite} = 10.39\text{cm}$$

Worked example - finding a missing angle:

So you've seen how to find a missing side, but you also need to be able to use trigonometry to find a missing angle in a right-angled triangle.



Step 1: Make sure you label the sides first (as above) and write out SOH CAH TOA to help you.

Step 2: Select which ratio to use (Sine, Cosine, or Tangent) based on the sides you are working with. In this case we have been given the adjacent and hypotenuse - so we will use the Cosine ratio.

Step 3: Set up the Cosine formula and then insert the numbers for this question:

$$\cos x = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\cos x = \frac{7}{10}$$

Step 4: To rearrange this formula - to get x as the subject by itself - we need to use something called 'inverse cosine' (which appears as " \cos^{-1} "). This is simply a button on your calculator (press shift then cos). So, the formula now looks like this:

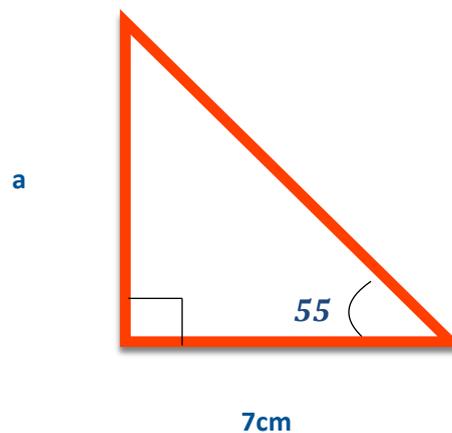
$$x = \cos^{-1}(0.7)$$
$$x = 45.57$$

We do the same thing if we're working with sine or tangent as well - \sin^{-1} or \tan^{-1} . Again, these are just buttons on your calculator (press shift and then 'sin' or 'tan')

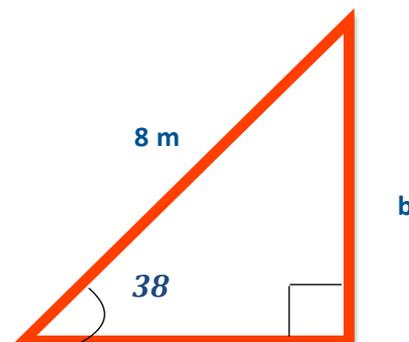
Practice questions

1. Find each of the unknown sides.

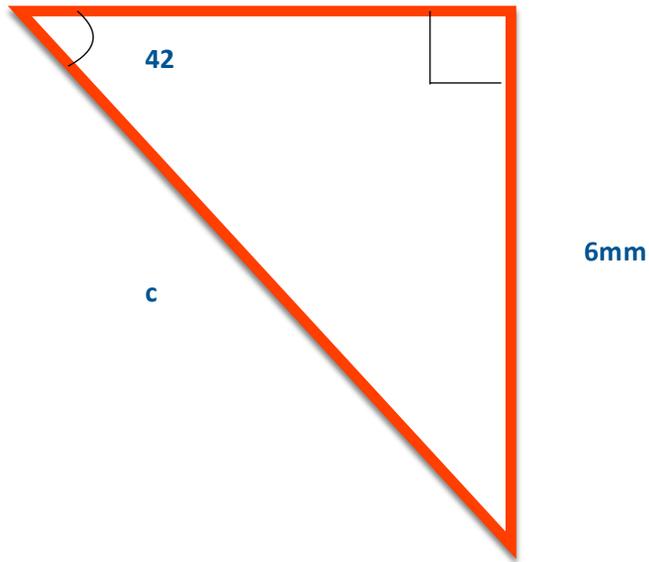
a)



b)

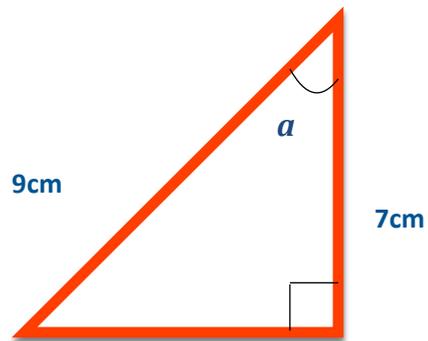


c)

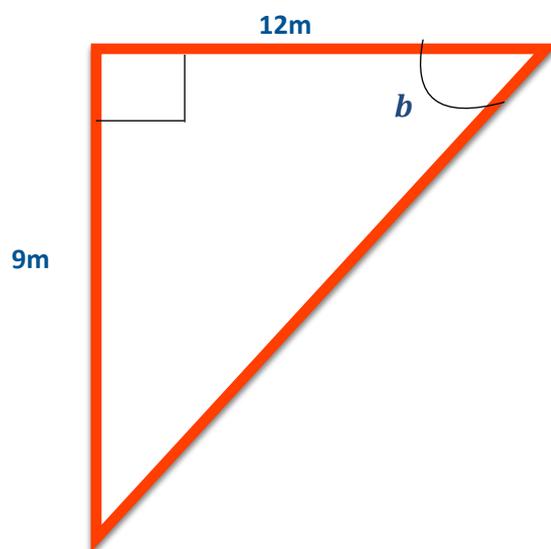


2. Now find the unknown angles on the following triangles.

a)



b)



c)

